Q1

i

Method 1

Find a^2 (by multiplying √8 + 2 by itself)

$$a^2 = (\sqrt{8}+2)(\sqrt{8}+2)$$
$$= \sqrt{8}\times\sqrt{8}+\sqrt{8}\times 2+2\times\sqrt{8}+2\times 2$$

[1]

Simplify the terms (using the surd law $\sqrt{a}\times\sqrt{a}=(\sqrt{a})^2=a$ for the first term)

$$a^2 = 8+2\sqrt{8}+2\sqrt{8}+4$$
$$= 12+4\sqrt{8}$$

Find b^2 (by multiplying √8 - 2 by itself) and do the same steps as above

$$b^2 = (\sqrt{8}-2)(\sqrt{8}-2)$$
$$= \sqrt{8}\times\sqrt{8}-\sqrt{8}\times 2-2\times\sqrt{8}+2\times 2$$
$$= 8-2\sqrt{8}-2\sqrt{8}+4$$
$$= 12-4\sqrt{8}$$

[1]

Find $a^2 - b^2$ by subtracting the two results above (using brackets around the results)

Collect like terms

$$a^2-b^2 = (12+4\sqrt{8})-(12-4\sqrt{8})$$
$$= 12+4\sqrt{8}-12+4\sqrt{8}$$
$$= 8\sqrt{8}$$
$$= 8\sqrt{8}$$

[1]

The result must be given in the form $c\sqrt{2}$ where c is an integer (whole number)

Write $\sqrt{8}$ as $2\sqrt{2}$ (from $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\sqrt{2}=2\sqrt{2}$)

$$a^2-b^2 = 8\times 2\sqrt{2}$$
$$= 16\sqrt{2}$$

$16\sqrt{2}$ (where c = 16) [1]

Method 2

Use the "difference of two squares" to factorise a^2 - b^2

$$a^2-b^2 = (a+b)(a-b)$$

[1]

Substitute a = √8 + 2 and b = √8 - 2 into the expression and collect like terms

$$a^2-b^2 = (\sqrt{8}+2+\sqrt{8}-2)(\sqrt{8}+2-(\sqrt{8}-2))$$
$$= (2\sqrt{8})(\sqrt{8}+2-\sqrt{8}+2)$$
$$= (2\sqrt{8})(4)$$
$$= 8\sqrt{8}$$

[1]

The result must be given in the form $c\sqrt{2}$ where c is an integer (whole number)

Write $\sqrt{8}$ as $2\sqrt{2}$ (from $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\sqrt{2}=2\sqrt{2}$)

The result must be given in the form $c\sqrt{2}$ where c is an integer (whole number)
Write $\sqrt{8}$ as $2\sqrt{2}$ (from $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\sqrt{2}=2\sqrt{2}$)

$$a^2-b^2=8\times 2\sqrt{2}$$

[1]

Simplify the final answer (using $8 \times 2 = 16$)

$16\sqrt{2}$ (where $c = 16$) [1]

Q2

Rationalise the denominator (by multiplying top-and-bottom by $\sqrt{2}+1$)
($-\sqrt{2}-1$ would also work)

$$\frac{6-\sqrt{8}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

[1]

Multiply the numerators together and multiply the denominators together

$$\frac{(6-\sqrt{8})(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

Expand the brackets

$$\frac{6\times\sqrt{2}+6\times 1-\sqrt{8}\times\sqrt{2}-\sqrt{8}\times 1}{\sqrt{2}\times\sqrt{2}+\sqrt{2}\times 1-1\times\sqrt{2}-1\times 1}$$

[1]

Simplify the terms (using the surd laws $\sqrt{a}\times\sqrt{a}=(\sqrt{a})^2=a$ and $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$)

$$\frac{6\sqrt{2}+6-\sqrt{16}-\sqrt{8}}{2-1}$$
$$=\frac{6\sqrt{2}+6-4-\sqrt{8}}{1}$$
$$=6\sqrt{2}+2-\sqrt{8}$$

The final answer must be in the form $a+b\sqrt{2}$ where a and b are integers (whole numbers)
Write $\sqrt{8}$ as $2\sqrt{2}$ (from $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\sqrt{2}=2\sqrt{2}$) and collect like terms

$$=6\sqrt{2}+2-2\sqrt{2}$$
$$=4\sqrt{2}+2$$

This can be rewritten as $2+4\sqrt{2}$, which is the correct form of the answer

$2+4\sqrt{2}$ [1]

Q3

3

Method 1

Multiply top-and-bottom by √2

$$\frac{1}{1+\frac{1}{\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)}$$
$$= \frac{\sqrt{2}}{\sqrt{2}+1}$$

[1]

Rationalise the denominator (by multiplying top-and-bottom by √2 - 1)
(-√2 + 1 would also work)

$$\frac{\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

[1]

Multiply the numerators together and multiply the denominators together

$$\frac{\sqrt{2}\times\sqrt{2}-\sqrt{2}\times 1}{\sqrt{2}\times\sqrt{2}+\sqrt{2}\times 1-1\times\sqrt{2}-1\times 1}$$

Multiply the numerators together and multiply the denominators together

$$\frac{\sqrt{2}\times\sqrt{2}-\sqrt{2}\times 1}{\sqrt{2}\times\sqrt{2}+\sqrt{2}\times 1-1\times\sqrt{2}-1\times 1}$$

Simplify the terms (using the surd law $\sqrt{a}\times\sqrt{a}=\left(\sqrt{a}\right)^2=a$)

$$\frac{2-\sqrt{2}}{2-1}$$
$$=\frac{2-\sqrt{2}}{1}$$

The final answer is 2 - √2

$\mathbf{2-\sqrt{2}}$ [1]

Method 2

Rationalise the denominator of $\frac{1}{\sqrt{2}}$ (by multiplying top-and-bottom by √2)

Use that √2 × √2 = 2

$$\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$
$$=\frac{\sqrt{2}}{2}$$

Write out the fraction using this result

$$\frac{1}{1+\frac{\sqrt{2}}{2}}$$

[1]

Rationalise the denominator (by multiplying top-and-bottom by $1-\frac{\sqrt{2}}{2}$)

(Another way is to first multiply top-and-bottom by 2, then rationalise the denominator)

$$\frac{1}{\left(1+\frac{\sqrt{2}}{2}\right)}\times\frac{\left(1-\frac{\sqrt{2}}{2}\right)}{\left(1-\frac{\sqrt{2}}{2}\right)}$$

[1]

Expand the denominator (for example, using the difference of two squares, $(a+b)(a-b)=a^2-b^2$)

$$\frac{1-\frac{\sqrt{2}}{2}}{1-\frac{2}{4}}$$

$$=\frac{1-\frac{\sqrt{2}}{2}}{\frac{1}{2}}$$

Dividing by a half is the same as multiplying by 2 ($\div\frac{1}{2}$ means $\times\frac{2}{1}$)

Dividing by a half is the same as multiplying by 2 ($\div\frac{1}{2}$ means $\times\frac{2}{1}$)

$$=2\left(1-\frac{\sqrt{2}}{2}\right)$$

$$=2-\sqrt{2}$$

$\mathbf{2-\sqrt{2}}$ [1]

Q4

4

Substitute the values of a and b into the expression

$$(a-b)(a+b)=((\sqrt{8}+4)-(\sqrt{8}-4))\times((\sqrt{8}+4)+(\sqrt{8}-4))$$

Simplify the expressions

$$(\sqrt{8}-\sqrt{8}+4--4)\times(\sqrt{8}+\sqrt{8}+4-4)=(8)\times(2\sqrt{8}) = 16\sqrt{8}$$

Any equivalent to $16\sqrt{8}$ e.g. $32\sqrt{2}$ [2]

We need to rewrite in the form $y\sqrt{4y}$

Note that this is not as easy as finding the simplest form of the surd

Consider writing $y\sqrt{4y}$ in a different way, which may be easier

$$y\sqrt{4y} = y\times\sqrt{4}\times\sqrt{y} = 2y\sqrt{y}$$

This is more similar to our answer of $16\sqrt{8}$, and in this case $y=8$

As the two forms are equivalent, and $y=8$, we could write the answer using $y\sqrt{4y}$

$\mathbf{8\sqrt{4\times 8}}$ [1]

Q5

5

Multiply the top and bottom of the fraction by $a+\sqrt{4b}$

$$\frac{a+\sqrt{4b}}{a-\sqrt{4b}} \times \frac{a+\sqrt{4b}}{a+\sqrt{4b}} = \frac{a^2+a\sqrt{4b}+a\sqrt{4b}+\left(\sqrt{4b}\right)^2}{a^2-a\sqrt{4b}+a\sqrt{4b}-\left(\sqrt{4b}\right)^2}$$

Multiply top and bottom by $a+\sqrt{4b}$ [1]

Simplify

$$\frac{a^2+2a\sqrt{4b}+4b}{a^2-4b}$$

Simplify denominator [1]

The surd on the top of the fraction can be simplified

$$2a\sqrt{4b} = 2a\sqrt{4}\sqrt{b} = 2a\times 2\times\sqrt{b} = 4a\sqrt{b}$$

Rewrite using the simplified surd

$$\mathbf{\frac{a^2+4a\sqrt{b}+4b}{a^2-4b}}$$ [1]

Could also be written as $\frac{\left(a+2\sqrt{b}\right)^2}{a^2-4b}$

Q6

6

You need to rationalise the denominator. Multiply the top and bottom of the fraction by the denominator with the sign in the middle changed.

$$\frac{2}{\left(6-3\sqrt{2}\right)} \times \frac{\left(6+3\sqrt{2}\right)}{\left(6+3\sqrt{2}\right)}$$

[1]

Expand the brackets. Simplify the bottom.

$$\frac{12+6\sqrt{2}}{36+18\sqrt{2}-18\sqrt{2}-(9\times 2)}$$
$$=\frac{12+6\sqrt{2}}{36-18}$$
$$=\frac{12+6\sqrt{2}}{18}$$

[1]

Cancel out the common factor of 6.

$$\mathbf{\frac{2+\sqrt{2}}{3}}$$ [1]

Q7

7

You need to rationalise the denominator. Multiply the top and bottom of the fraction by the denominator with the sign in the middle changed.

$$\frac{8}{(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$$

[1]

Expand the brackets. Simplify the bottom.

$$\frac{8\sqrt{5}+8}{5-\sqrt{5}+\sqrt{5}-1} = \frac{8\sqrt{5}+8}{4}$$

Cancel out the common factor of 4.

$$2\sqrt{5}+2$$

[1]

The question asks for the first term to be written as a square root so use the fact that $2=\sqrt{4}$.

$$\sqrt{4}\times\sqrt{5}+2$$

Use the fact that $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$.

$\sqrt{\mathbf{20}}+\mathbf{2}$ [1]

Q8

8

The area of a rectangle is found by multiplying the two perpendicular lengths together. One of the lengths can be found by dividing the area by the other length.

$$\text{Length} = \frac{18}{\sqrt{7}+1}$$

You need to rationalise the denominator. Multiply the top and bottom of the fraction by the denominator with the sign in the middle changed.

$$\frac{18}{(\sqrt{7}+1)} \times \frac{(\sqrt{7}-1)}{(\sqrt{7}-1)}$$

[1]

Expand the brackets. Simplify the bottom.

$$\frac{18\sqrt{7}-18}{7-\sqrt{7}+\sqrt{7}-1} = \frac{18\sqrt{7}-18}{6}$$

[1]

Cancel out the common factor of 6.

$(\mathbf{3}\sqrt{\mathbf{7}}-\mathbf{3})$ **cm** [1]

Q9

9

First simplify √8.

$$\sqrt{8}=\sqrt{4}\times\sqrt{2}=2\sqrt{2}$$

Rationalise the denominator by multiplying the top and bottom by $\sqrt{2}+1$.

$$\frac{(4+2\sqrt{2})}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{(4+2\sqrt{2})(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

[1]

Expand and simplify the expressions.

$$\frac{4\sqrt{2}+4+(2\times 2)+2\sqrt{2}}{(\sqrt{2})^2+\sqrt{2}-\sqrt{2}-1} = \frac{6\sqrt{2}+4+4}{2-1} = \frac{8+6\sqrt{2}}{1}$$

[1]

$\mathbf{8}+\mathbf{6}\sqrt{\mathbf{2}}$ [1]

Q10

10

To rationalise the denominator we multiply by $\frac{\sqrt{5}+1}{\sqrt{5}+1}$

$$\frac{4+2\sqrt{5}}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$

[1]

Multiply the numerators, and multiply the denominators
Make sure to multiply all terms by each other, e.g. 4 must be multiplied by both $\sqrt{5}$ and 1, and the same is true for $2\sqrt{5}$

$$\frac{4+2\sqrt{5}}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}=\frac{4\sqrt{5}+4+2\sqrt{5}\sqrt{5}+2\sqrt{5}}{\sqrt{5}\sqrt{5}+\sqrt{5}-\sqrt{5}-1}$$

Correct numerator [1]
Correct denominator [1]

Simplify where possible

$$\frac{4\sqrt{5}+4+(2\times5)+2\sqrt{5}}{5-1}=\frac{6\sqrt{5}+14}{4}$$

Divide the numerator and denominator by 2

$\mathbf{\frac{3\sqrt{5}+7}{2}}$ [1]